E 215 (Pages : 3) Reg. No.....

Name.....

C.B.C.S.S. B.Sc. DEGREE EXAMINATION, NOVEMBER 2009

First Semester

Core Course—Mathematics—FOUNDATION OF MATHEMATICS

(Common for B.Sc. Mathematics — (Model I) B.Sc. Mathematics — (Model II) and B.Sc. Computer Applications (Triple Main)

Time : Three Hours Total Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*

A bunch of four questions has weight **one.**

I. 1 Find two sets A and B such that $A \in B$ and $A \subseteq B$.

2 Why $f(x) = \sqrt{x}$ is not a function from $\mathbb{R}$ to $\mathbb{R}$.

3 Let S = {– 1, 0, 2, 4, 7}. Find $f(s)$ if $f(x) = \left\lceil \frac{x}{5} \right\rceil$.

4 Write all the ordered pairs in the relation $R = \{(a,b) | a \text{ divides } b\}$ on the set {1, 2, 3, 4, 5, 6}.

II. 5 Let R be the relation on the set of integers such that aRb iff $a = b$ or $a = -b$. What is the equivalence class of 2 for this equivalence relation ?

6 Let $R_1 = \{(a,b) \in \mathbb{R}^2 | a > b\}$

$R_2 = \{(a,b) \in \mathbb{R}^2 | a \geq b\}$.

Find $R_1 - R_2$.

7 Define a transitive relation on a set A.

8 Let S be a set. Determine whether there is a greatest element and a least element in the poset $(P(s), \subseteq)$.

III. 9 State the contrapositive of the implication :

If it snows tonight, then I will stay at home.

10 Let $p(x)$ be the statement "$x + 1 > x$". What is the truth value of $\forall x\, P(x)$, where the universe of discourse consists of all real numbers ?

11 Define a vacous proof.

12 Let p and q be the propositions

p : It is below freezing

q : It is snowing.

Write the proposition "It is below freezing but not snowing" using p and q and logical connectives.

Turn over

V. 13 Find the number of integers less than n and prime to it when $n = 1024$.

14 Find the form of x, if $2x \equiv 3 \pmod 7$

15 Show that every number is one of the forms $5n$, $5n \pm 1$, $5n \pm 2$.

16 If the greatest common divisor of a and b is d, then what is the greatest common divisor of $\frac{a}{m}$ and $\frac{b}{m}$ where $m \neq 0$?

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Let f be the function from Z to Z with $f(x) = x^2$. Is f invertible ? Justify your answer.

18 Let R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)} and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2)}. Find $S \circ R$.

19 Prove that the relation R defined on the set of real numbers defined by aRb if and only if $a - b$ is an integer is an equivalence relation.

20 Construct a truth table for the compound proposition :

$(p \wedge q) \rightarrow (p \vee q)$.

21 Suppose that the universe of discourse of the propositional function $P(x)$ consist of the integers 0, 1, 2, 3 and 4. Write the proposition $\exists x\, P(x)$ using disjunctions, conjunctions and negations.

22 State Wilson's theorem.

23 If $a \equiv b \pmod n$ and $a' \equiv b' \pmod n$, prove that $aa' \equiv bb' \pmod n$.

24 Using Fermat's theorem, prove that if p is a prime and a is any number, then $a^p - a$ is divisible by p.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 Let f be a function from A to B. Let S and T be subsets of B. Show that $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$.

26 Let R_1 and R_2 be relations on a set A represented by the matrices :

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrices that represent (i) $R_1 \cup R_2$; (ii) $R_2 \circ R_1$.

27 Show that the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

28 Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

29 Show that $2^{2n} - 3n - 1$ is divisible by 9.

30 Prove that $\lfloor 28 + 233$ is divisible by 899.

$(4 \times 2 = 8)$

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 (a) How many relations are there on a set with n elements ?

(b) How many reflexive relations are there on a set with n elements :

Justify your answers.

32 Show that the following statements are equivalent :

P_1 : n is an even integer

P_2 : $n - 1$ is an odd integer

P_3 : n^2 is an even integer.

33 Prove that if m is prime to n, then $\phi(mn) = \phi(m)\,\phi(n)$.

$(2 \times 4 = 8)$